B.Sc Physics Part-III

Paper-V, Group A

Mathematical Physics

Harmonic Functions:

~~If we take f(z) = u+iv to be~~

Let $f(z) = u+iv$ is an analytic function in some region of the z-plane. The Cauchy-Riemann equations are given by

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{--- (1)}$$

$$\text{and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad \text{--- (2)}$$

In earlier class notes we have seen how to get the above conditions. Next, we differentiate eq. (1) w.r.t. x and eq. (2) w.r.t. y and obtain

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x \partial y} \quad \text{--- (3)}$$

$$\text{and } \frac{\partial^2 u}{\partial y^2} = -\frac{\partial^2 v}{\partial y \partial x} \quad \text{--- (4)}$$

Let us assume that $\frac{\partial^2 v}{\partial x \partial y} = \frac{\partial^2 v}{\partial y \partial x}$ and adding eqs. (3) and (4), we obtain

$$\boxed{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0} \quad \text{--- (5)}$$

similarly following the same procedure as above, we differentiate eq. (1) w.r.to y and eq. (2) w.r.to. x and substracting, we get

$$\boxed{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0} \quad \text{--- (6)}$$

Eq. (5) & (6) shows that functions u and v satisfy the Laplace's equation in two variables and therefore they are called as Harmonic functions.